

$B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$: Detecting and Discriminating New Physics in B_s^0 - \bar{B}_s^0 Mixing

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If the weak phase of B_s^0 - \bar{B}_s^0 mixing ($2\beta_s$) is found to be significantly different from zero, this is a clear signal of new physics (NP). However, if such a signal is found, we would like an unambiguous determination of $2\beta_s$ in order to ascertain which NP models could be responsible. In addition, in the presence of NP, the width difference $\Delta\Gamma_s$ between the two B_s mass eigenstates can be positive or negative, and ideally this sign ambiguity should be resolved experimentally. Finally, in order to see if the NP is contributing to Γ_{12}^s in addition to M_{12}^s , the precise measurement of $|\Gamma_{12}^s|$ is crucial. In this paper, we consider several different methods of measuring B_s^0 - \bar{B}_s^0 mixing using two- and three-body decays with $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$ transitions. We find that the most promising of these is a time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$. With these decays, all of the above issues can be addressed, and the measurement of the weak phase γ is also possible. We also note that, with all three-body decays it is possible to resolve the sign ambiguity of $\Delta\Gamma_s$ even without determining CP phase ϕ_s .

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I. INTRODUCTION

Over the past several years, a number of discrepancies with the predictions of the standard model (SM) have been observed in B decays, intriguingly all in $\bar{b} \rightarrow \bar{s}$ transitions. Some examples are: (i) in $B \rightarrow \pi K$ decays, it is difficult to account for all the experimental measurements within the SM [1, 2], (ii) the values of the B_d^0 - \bar{B}_d^0 mixing phase $\sin 2\beta$ obtained from different penguin-dominated $\bar{b} \rightarrow \bar{s}$ channels tend to be systematically smaller than that obtained from $B_d^0 \rightarrow J/\psi K_S$ [3], (iii) the fractions of transversely- and longitudinally-polarized decays in $B \rightarrow \phi K^*$ (f_T and f_L , respectively) are observed to be roughly equal [4], in contrast to the naive expectation that $f_T/f_L \ll 1$, (iv) the differential forward-backward asymmetry of leptons in the exclusive decay $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$ is found to differ from the SM expectations in both the low- and high- q^2 regions (q^2 is the dilepton invariant mass) [5, 6].

In light of this, it is particularly important to study $\bar{b} \rightarrow \bar{s}$ transitions and look for new-physics (NP) effects. Now, if NP is present in $\Delta B = 1$ $\bar{b} \rightarrow \bar{s}$ decays, it would be highly unnatural for it not to also affect the $\Delta B = 2$ transition, in particular B_s^0 - \bar{B}_s^0 mixing. In order to see where NP can enter, we briefly review the mixing. In the B_s system, the mass eigenstates B_L and B_H (L and H indicate the light and heavy states, respectively) are admixtures of the flavor eigenstates B_s^0 and \bar{B}_s^0 :

$$\begin{aligned} |B_L\rangle &= p |B_s^0\rangle + q |\bar{B}_s^0\rangle, \\ |B_H\rangle &= p |B_s^0\rangle - q |\bar{B}_s^0\rangle, \end{aligned} \quad (1)$$

with $|p|^2 + |q|^2 = 1$. As a result, the initial flavor eigenstates oscillate into one another according to the Schrödinger equation

$$i \frac{d}{dt} \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix} = \left(M^s - i \frac{\Gamma^s}{2} \right) \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix}, \quad (2)$$

where $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$ correspond respectively to the dispersive and absorptive parts of the mass matrix. The off-diagonal elements, $M_{12}^s = M_{21}^{s*}$ and $\Gamma_{12}^s = \Gamma_{21}^{s*}$, are generated by B_s^0 - \bar{B}_s^0 mixing. We define

$$\Gamma_s \equiv \frac{\Gamma_H + \Gamma_L}{2}, \quad \Delta M_s \equiv M_H - M_L, \quad \Delta\Gamma_s \equiv \Gamma_L - \Gamma_H. \quad (3)$$

Expanding the mass eigenstates, we find, to a very good approximation [7],

$$\begin{aligned} \Delta M_s &= 2|M_{12}^s|, \\ \Delta\Gamma_s &= 2|\Gamma_{12}^s| \cos \phi_s, \\ \frac{q}{p} &= e^{-2i\beta_s} \left[1 - \frac{a}{2} \right], \end{aligned} \quad (4)$$

where $\phi_s \equiv \arg(-M_{12}^s/\Gamma_{12}^s)$ is the CP phase in $\Delta B = 2$ transitions, and $2\beta_s = \arg(M_{12}^s)$. In Eq. (4) the small expansion parameter a is given by

$$a = \frac{\Gamma_{12}^s}{M_{12}^s} \sin \phi_s. \quad (5)$$

This is expected to be $\ll 1$, and hence can be neglected in the definition of q/p . It is also important to note that the sign of $\Delta\Gamma_s$ is equal to the sign of $\cos \phi_s$, and in the case where there is no NP in Γ_{12}^s , the CP phase $\phi_s = -2\beta_s$.

The precise measurement of ΔM_s determines $|M_{12}^s|$ [8]. However, because of hadronic uncertainties, the SM prediction for ΔM_s is not very precise – in Ref. [9], it is noted that the theoretical uncertainties still allow new-physics contributions to $|M_{12}^s|$ of order 20%. In addition, Γ_{12}^s can be calculated from the absorptive part of the B_s^0 - \bar{B}_s^0 mixing box diagram, leading to $\Delta\Gamma_s$. Unlike the B_d system, where $\Delta\Gamma_d$ is negligibly small, in the B_s system $\Delta\Gamma_s$ is expected to be reasonably large, which leads to certain advantages for the search for CP-violating effects in the B_s system over that of B_d system. The updated SM predictions of the width difference and the CP phase ϕ_s are given by [9]

$$\begin{aligned} \Delta\Gamma_s^{SM} &\simeq 2|\Gamma_{12}^s| = 0.087 \pm 0.021 \text{ ps}^{-1}, \\ \phi_s &\approx 0.22^\circ. \end{aligned} \quad (6)$$

Although the SM predictions for ΔM_s and $\Delta\Gamma_s$ are not precise, the SM does predict that $2\beta_s \simeq 0$, which makes it a good observable to use in the search for NP. Consider first the case where the NP contributes only to M_{12}^s . If one measures a value for $2\beta_s$ that is significantly different from zero, this will indicate NP in B_s^0 - \bar{B}_s^0 mixing (in M_{12}^s). However, to cover all bases, one more step must be done. Suppose that NP is present, but it produces $2\beta_s = 180^\circ$. Now indirect CP violation, which measures $\sin 2\beta_s$, will not give a signal. But one can still detect the NP by measuring the sign of $\Delta\Gamma_s$ – in the SM, $\Delta\Gamma_s > 0$, while it is < 0 if $2\beta_s = 180^\circ$. Also, even if NP is discovered through indirect CP violation (i.e. $\sin 2\beta_s \neq 0$), this only determines $2\beta_s$ up to a twofold discrete ambiguity. Since the sign of $\cos 2\beta_s$ can be determined by the sign of $\Delta\Gamma_s$, the knowledge of this sign is one possibility to remove this discrete ambiguity. Alternatively, one may try to find a method which allows a direct determination of $2\beta_s$ without any ambiguity. Now suppose that the NP also contributes to Γ_{12}^s . Since there is NP in M_{12}^s , its presence can be detected as above. But now the twofold ambiguity in $2\beta_s$ cannot be removed from the knowledge of the sign of $\Delta\Gamma_s$, since this only determines the sign of $\cos \phi_s$ (and not $\cos 2\beta_s$). Thus, for the case where there is NP in Γ_{12}^s , one must find another way to remove the twofold ambiguity in $2\beta_s$.

The CDF [10] and DØ [11] collaborations have measured the CP asymmetry in $B_s^0 \rightarrow J/\psi\phi$, and found a hint for indirect CP violation. In general, this result is interpreted as evidence for a nonzero value of $2\beta_s^{\psi\phi}$, and the contributions of various NP models to the B_s mixing phase have been explored [12–18]. It has also been pointed out that NP in the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$ could also play an important role [19]. Recently CDF and DØ updated their measurements of the CP-violating phase. The 68% C.L. allowed ranges are [20, 21]

$$\begin{aligned} 2\beta_s^{\psi\phi} &\in [2.3^\circ, 59.6^\circ] \cup [123.8^\circ, 177.6^\circ], & \text{CDF}, \\ &\in [9.7^\circ, 52.1^\circ] \cup [127.9^\circ, 170.3^\circ], & \text{DØ}. \end{aligned} \quad (7)$$

Most of the values of $2\beta_s^{\psi\phi}$ here suggest NP. $2\beta_s^{\psi\phi}$ is obtained with the twofold ambiguity $2\beta_s^{\psi\phi} \leftrightarrow \pi - 2\beta_s^{\psi\phi}$, and at present there is no preference for either of the two solutions. As mentioned above, the possibility of NP in the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$ cannot be ruled out, so that the phase $2\beta_s^{\psi\phi}$ extracted from $B_s^0 \rightarrow J/\psi\phi$ should not necessarily be taken as purely a mixing phase. It is therefore worthwhile to look for a process in which NP in the decay can essentially be neglected, and which permits the determination of $2\beta_s$ without any ambiguity. If the measured value of $2\beta_s$ is found to be significantly different from that in $B_s^0 \rightarrow J/\psi\phi$, it will be clear signal of NP in $\bar{b} \rightarrow \bar{s}c\bar{c}$.

In addition, the DØ Collaboration recently found a large CP asymmetry in the like-sign dimuon signal, which they attribute primarily to a_{SL}^s , the semileptonic CP asymmetry in $B_s^0 \rightarrow X_s\mu\nu$ [22, 23]. Now, the DØ result is less than 2σ away from zero and consequently to an excellent approximation also about 2σ away from the SM prediction ($a_{SL}^{s,SM} \approx 2 \times 10^{-5}$) [9]. Still, NP in B_s^0 - \bar{B}_s^0 mixing can explain the result (for example, see Ref. [24]). However, if one wishes to reproduce the central value of a_{SL}^s , one requires NP specifically in Γ_{12}^s [25, 26]. There are NP models that can contribute to Γ_{12}^s through the decay $b \rightarrow s\tau^+\tau^-$ [27, 28], and a significant enhancement of its magnitude over that of the SM [9] is possible. Furthermore, the possibility of NP effects in Γ_{12}^s through the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$ cannot be ruled out [26, 27].

We therefore see that there are some hints of NP in the B_s system, but nothing definitive yet. Thus, it is important to look for additional methods of probing NP in B_s^0 - \bar{B}_s^0 mixing. Ideally, the new method(s) would allow an unambiguous determination of the mixing phase $2\beta_s$. Also useful are methods which remove the sign ambiguity in $\Delta\Gamma_s$ even without providing any direct information on the CP phases ϕ_s or $2\beta_s$. Finally, if NP is present in the mixing, we would like

to know if it contributes to Γ_{12}^s in addition to M_{12}^s . Hence, along with the removal of the sign ambiguity in $\Delta\Gamma_s$, independent and unbiased measurements of $|\Gamma_{12}^s|$ and ϕ_s are essential.

Several years ago, the two-body decays $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp, D_s^{*\pm} K^\mp, \dots$ were examined with the idea of extracting weak phases [29]. Because the final state is accessible to both B_s^0 and \bar{B}_s^0 mesons, a mixing-induced indirect CP asymmetry occurs. Using this, and assuming that $\Delta\Gamma_s$ is sizeable, the conclusion of Ref. [29] is that one can measure the phase $2\beta_s + \gamma$ with a twofold discrete ambiguity, and that this ambiguity can be removed if factorization is assumed. However, if there is NP in B_s^0 - \bar{B}_s^0 mixing, $\Delta\Gamma_s < 0$ is allowed as well. This implies that, in fact, $2\beta_s + \gamma$ can be obtained with a fourfold discrete ambiguity (or twofold if factorization is assumed).

In Ref. [30] it was shown that the sign ambiguity in $\Delta\Gamma_s$ can be removed using $B_s^0 \rightarrow D_s^\pm K^\mp$ decays. Although the method does not allow a direct determination of the phase $2\beta_s$, it does discriminate between the two solutions with $\cos 2\beta_s > 0$ and $\cos 2\beta_s < 0$, which then determines the sign of $\Delta\Gamma_s$. However, the method is based on several assumptions: (i) the weak phase γ is taken from the B -factory measurements, (ii) factorization is assumed, i.e. the strong phase is taken to be $\simeq 0$, and (iii) the SM-predicted value of Γ_{12}^s has been used in the analysis.

In 1991, the decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 \phi$, where D_{CP}^0 is a neutral D -meson CP-eigenstate, were proposed to extract the CKM angle γ with a twofold ambiguity [31, 32]. However, these methods assumed that the phase $2\beta_s$ is approximately zero (or known). The current experimental data [see Eq. (7)] is not completely in favor of this assumption – there is the possibility that $2\beta_s$ can be significantly different from zero. In addition, at present $2\beta_s$ is measured with a twofold ambiguity, which adds a further discrete ambiguity to the determination of γ .

We therefore see that previous analyses of two-body B decays only partially probe NP in B_s^0 - \bar{B}_s^0 mixing – $2\beta_s$ is, in general, not determined unambiguously, the sign ambiguity in $\Delta\Gamma_s$ is generally unresolved, and the possibility of NP affecting Γ_{12}^s has not been considered. In this paper we go beyond the previous analyses to explore all of these issues.

In Sec. II we review the two-body decays. In particular, in Sec. II C, we update the analysis of $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 \phi$, considering both $\Delta\Gamma_s > 0$ and $\Delta\Gamma_s < 0$. In Sec. III, we present the Dalitz-plot analyses of three-body decays. In particular, in Sec. III C, we focus on $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0, D_s^\pm \pi^\mp K^0, \dots$, using the interference between the different intermediate resonant decays to provide additional information. And in Sec. III D, we show that a much greater improvement can be obtained by performing a time-dependent Dalitz-plot analysis of the decay $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$. Finally, in Sec. IV, we present a possible way to determine $\Delta\Gamma_s$, or equivalently $|\Gamma_{12}^s|$ and ϕ_s , using three-body decays. We conclude in Sec. V.

II. TWO-BODY DECAYS

A. $B_s^0(\bar{B}_s^0) \rightarrow f, \bar{f}$

Consider a final state f , not necessarily a CP eigenstate, to which both B_s^0 and \bar{B}_s^0 can decay. In the presence of B_s^0 - \bar{B}_s^0 mixing, the time-dependent decay rates are given by [33]

$$\begin{aligned} \Gamma(B_s^0(t) \rightarrow f) &\sim \frac{1}{2} e^{-\Gamma_s t} \left\{ (|A_f|^2 + |\bar{A}_f|^2) \cosh(\Delta\Gamma_s t/2) + (|A_f|^2 - |\bar{A}_f|^2) \cos \Delta m_s t \right. \\ &\quad \left. - 2 \sinh(\Delta\Gamma_s t/2) \operatorname{Re} \left[\frac{q}{p} A_f^* \bar{A}_f \right] - 2 \sin \Delta m_s t \operatorname{Im} \left[\frac{q}{p} A_f^* \bar{A}_f \right] \right\}, \\ \Gamma(\bar{B}_s^0(t) \rightarrow f) &\sim \frac{1}{2} e^{-\Gamma_s t} \left\{ (|A_f|^2 + |\bar{A}_f|^2) \cosh(\Delta\Gamma_s t/2) - (|A_f|^2 - |\bar{A}_f|^2) \cos \Delta m_s t \right. \\ &\quad \left. - 2 \sinh(\Delta\Gamma_s t/2) \operatorname{Re} \left[\frac{q}{p} A_f^* \bar{A}_f \right] + 2 \sin \Delta m_s t \operatorname{Im} \left[\frac{q}{p} A_f^* \bar{A}_f \right] \right\}, \end{aligned} \quad (8)$$

where $A_f \equiv A(B_s^0 \rightarrow f)$, $\bar{A}_f \equiv A(\bar{B}_s^0 \rightarrow f)$, and $q/p = e^{-2i\beta_s}$. This yields

$$\begin{aligned} \Gamma(B_s^0(t) \rightarrow f) - \Gamma(\bar{B}_s^0(t) \rightarrow f) &\sim (|A_f|^2 + |\bar{A}_f|^2) e^{-\Gamma_s t} [C \cos \Delta m_s t - S \sin \Delta m_s t], \\ \Gamma(B_s^0(t) \rightarrow f) + \Gamma(\bar{B}_s^0(t) \rightarrow f) &\sim (|A_f|^2 + |\bar{A}_f|^2) e^{-\Gamma_s t} [\cosh(\Delta\Gamma_s t/2) - \mathcal{A}_{\Delta\Gamma} \sinh(\Delta\Gamma_s t/2)], \end{aligned} \quad (9)$$

where

$$C \equiv \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad S \equiv \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2}, \quad \mathcal{A}_{\Delta\Gamma} \equiv \frac{2 \operatorname{Re} \lambda}{1 + |\lambda|^2}, \quad \lambda \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (10)$$

The idea is that, by fitting the data corresponding to the difference (“tagged”) and sum (“untagged”) of decay rates to the four time-dependent functions given on the right-hand side of the equations in Eq. (9), the coefficients of these

functions can be obtained, from which C , S and $\mathcal{A}_{\Delta\Gamma}$ can be derived. However, there is a complication – in the presence of NP in $\Delta B = 2$ transitions, $\Delta\Gamma_s$ is unknown (though it is assumed to be reasonably large). Therefore, for the untagged combination, both $\Delta\Gamma_s$ and $\mathcal{A}_{\Delta\Gamma}$ must be found in the fit. Still, though this will determine $|\Delta\Gamma_s|$, its sign will remain unknown. The reason is that only the function $\sinh(\Delta\Gamma_s t/2)$ is sensitive to the sign of $\Delta\Gamma_s$, and it is multiplied by $\mathcal{A}_{\Delta\Gamma}$. Thus, any change in the sign of $\Delta\Gamma_s$ can be compensated for by changing the sign of $\mathcal{A}_{\Delta\Gamma}$. The bottom line is that any analysis which uses $\mathcal{A}_{\Delta\Gamma}$ will have a discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$.

Similarly,

$$\begin{aligned}\Gamma(B_s^0(t) \rightarrow \bar{f}) &\sim \frac{1}{2}e^{-\Gamma_s t} \left\{ (|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2) \cosh(\Delta\Gamma_s t/2) + (|A_{\bar{f}}|^2 - |\bar{A}_{\bar{f}}|^2) \cos \Delta m_s t \right. \\ &\quad \left. - 2 \sinh(\Delta\Gamma_s t/2) \operatorname{Re} \left[\frac{p}{q} \bar{A}_{\bar{f}}^* A_{\bar{f}} \right] + 2 \sin \Delta m_s t \operatorname{Im} \left[\frac{p}{q} \bar{A}_{\bar{f}}^* A_{\bar{f}} \right] \right\}, \\ \Gamma(\bar{B}_s^0(t) \rightarrow \bar{f}) &\sim \frac{1}{2}e^{-\Gamma_s t} \left\{ (|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2) \cosh(\Delta\Gamma_s t/2) - (|A_{\bar{f}}|^2 - |\bar{A}_{\bar{f}}|^2) \cos \Delta m_s t \right. \\ &\quad \left. - 2 \sinh(\Delta\Gamma_s t/2) \operatorname{Re} \left[\frac{p}{q} \bar{A}_{\bar{f}}^* A_{\bar{f}} \right] - 2 \sin \Delta m_s t \operatorname{Im} \left[\frac{p}{q} \bar{A}_{\bar{f}}^* A_{\bar{f}} \right] \right\},\end{aligned}\quad (11)$$

where $A_{\bar{f}} \equiv A(B_s^0 \rightarrow \bar{f})$ and $\bar{A}_{\bar{f}} \equiv A(\bar{B}_s^0 \rightarrow \bar{f})$. Then

$$\frac{\Gamma(B_s^0(t) \rightarrow \bar{f}) - \Gamma(\bar{B}_s^0(t) \rightarrow \bar{f})}{\Gamma(B_s^0(t) \rightarrow \bar{f}) + \Gamma(\bar{B}_s^0(t) \rightarrow \bar{f})} = \frac{\bar{C} \cos \Delta m_s t + \bar{S} \sin \Delta m_s t}{\cosh(\Delta\Gamma_s t/2) - \bar{\mathcal{A}}_{\Delta\Gamma} \sinh(\Delta\Gamma_s t/2)}, \quad (12)$$

where

$$\bar{C} \equiv \frac{1 - |\bar{\lambda}|^2}{1 + |\bar{\lambda}|^2}, \quad \bar{S} \equiv \frac{2 \operatorname{Im} \bar{\lambda}}{1 + |\bar{\lambda}|^2}, \quad \bar{\mathcal{A}}_{\Delta\Gamma} \equiv \frac{2 \operatorname{Re} \bar{\lambda}}{1 + |\bar{\lambda}|^2}, \quad \bar{\lambda} \equiv \frac{p}{q} \frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}. \quad (13)$$

B. $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp$

Consider the decay $B_s^0(\bar{B}_s^0) \rightarrow PP$ (P is a pseudoscalar), in which the final state contains a single c quark¹. Excluding those final states involving η 's, there are only two decays in which the B_s^0 and \bar{B}_s^0 amplitudes are of comparable size: $B_s^0(\bar{B}_s^0) \rightarrow D_s^- K^+$ and $B_s^0(\bar{B}_s^0) \rightarrow D_s^+ K^-$. The B_s^0 decays are mediated by color-allowed tree-level transitions $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$. Within the SM, the amplitudes take the form² (there is a minus sign associated with the \bar{u} quark)

$$\begin{aligned}A(B_s^0 \rightarrow D_s^- K^+) &= T', & A(B_s^0 \rightarrow D_s^+ K^-) &= -\tilde{T}' e^{i\gamma}, \\ A(\bar{B}_s^0 \rightarrow D_s^- K^+) &= \tilde{T}' e^{-i\gamma}, & A(\bar{B}_s^0 \rightarrow D_s^+ K^-) &= -T'.\end{aligned}\quad (14)$$

We have explicitly written the weak-phase dependence, while the diagrams contain strong phases. The magnitudes of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}^* V_{us}|$ and $|V_{ub}^* V_{cs}|$ have been absorbed into the diagrams T' and \tilde{T}' , respectively. (As this is a $\bar{b} \rightarrow \bar{s}$ transition, the diagrams are written with primes.)

Using the amplitudes of Eq. (14), one obtains [see Eqs. (10) and (13)]

$$\begin{aligned}C &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad S = -\frac{2|\lambda|}{1 + |\lambda|^2} \sin(2\beta_s + \gamma - \delta), \quad \mathcal{A}_{\Delta\Gamma} = \frac{2|\lambda|}{1 + |\lambda|^2} \cos(2\beta_s + \gamma - \delta), \\ \bar{C} &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad \bar{S} = \frac{2|\lambda|}{1 + |\lambda|^2} \sin(2\beta_s + \gamma + \delta), \quad \bar{\mathcal{A}}_{\Delta\Gamma} = \frac{2|\lambda|}{1 + |\lambda|^2} \cos(2\beta_s + \gamma + \delta),\end{aligned}\quad (15)$$

where $|\lambda| = \tilde{T}'/T'$ (defined to be positive) and δ is the strong-phase difference between \tilde{T}' and T' . $|\lambda|$ can be obtained from the measurement of C . Using this, S and $\mathcal{A}_{\Delta\Gamma}$ give $\sin(2\beta_s + \gamma - \delta)$ and $\cos(2\beta_s + \gamma - \delta)$, respectively. Thus,

¹ Much of the discussion in this subsection can be found in Ref. [29], except that here NP in $\Delta\Gamma_s$ is considered.

² In Ref. [26], it is shown that NP in the decays $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$ is strongly constrained. Such NP contributions are therefore neglected throughout this paper.

one obtains $2\beta_s + \gamma - \delta$ with no discrete ambiguity. Similarly, $2\beta_s + \gamma + \delta$ can be obtained with no discrete ambiguity from \bar{S} and $\bar{A}_{\Delta\Gamma}$. These can be combined to give the phases $(2\beta_s + \gamma, \delta)$ with a twofold ambiguity $[(2\beta_s + \gamma, \delta) \text{ or } (2\beta_s + \gamma + \pi, \delta + \pi)]$. This discrete ambiguity can be removed if one assumes factorization, which predicts δ to be near 0.

In fact, this is not quite correct. As discussed below Eq. (10), in the presence of NP in $B_s^0 - \bar{B}_s^0$ mixing there is an additional discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$. Thus, the two-body decays $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp$ permit the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity (or twofold if factorization is assumed).

Now, the value of γ can be taken from the independent measurements at the B -factories. One then obtains $2\beta_s$ with a fourfold ambiguity. Alternatively, since γ has not been measured in B_s decays, it can be kept with the aim of determining its value independently (this was the original purpose of Ref. [29].) We adopt this latter approach in much of the paper.

We therefore see that this method permits the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity (or twofold if factorization is assumed). It does not resolve the sign ambiguity in $\Delta\Gamma_s$, and says nothing about the possibility of NP affecting Γ_{12}^* . In order to address these remaining points, it is necessary to examine other methods. A first step involves the decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0\phi$, discussed in the next subsection.

C. $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0\phi$

Another pair of decays to which the method of the previous subsection can be applied is $B_s^0(\bar{B}_s^0) \rightarrow D^0\phi, \bar{D}^0\phi$. Here the decays are mediated by color-suppressed tree-level transitions. The amplitudes (of comparable size) are given by

$$\begin{aligned} A(B_s^0 \rightarrow D^0\phi) &= -C_1^\phi e^{i\gamma}, & A(B_s^0 \rightarrow \bar{D}^0\phi) &= C_2^\phi, \\ A(\bar{B}_s^0 \rightarrow \bar{D}^0\phi) &= C_1^\phi e^{-i\gamma}, & A(\bar{B}_s^0 \rightarrow D^0\phi) &= -C_2^\phi. \end{aligned} \quad (16)$$

By measuring the time dependence of the decays, one can obtain $S, \bar{S}, A_{\Delta\Gamma}$ and $\bar{A}_{\Delta\Gamma}$ as given in Eqs. (10) and (13). Using these observables we define

$$\begin{aligned} \sin(2\beta_s + \gamma + \delta_\phi) &= -\frac{1 + |\lambda|^2}{2|\lambda|} S \equiv S_D, & \sin(2\beta_s + \gamma - \delta_\phi) &= \frac{1 + |\lambda|^2}{2|\lambda|} \bar{S} \equiv \bar{S}_D, \\ \cos(2\beta_s + \gamma + \delta_\phi) &= \frac{1 + |\lambda|^2}{2|\lambda|} A_{\Delta\Gamma} \equiv A_{\Delta\Gamma}^D, & \cos(2\beta_s + \gamma - \delta_\phi) &= \frac{1 + |\lambda|^2}{2|\lambda|} \bar{A}_{\Delta\Gamma} \equiv \bar{A}_{\Delta\Gamma}^D, \end{aligned} \quad (17)$$

with $\delta_\phi = \arg(C_1^\phi/C_2^\phi)$. The method of the previous subsection then allows us to obtain $2\beta_s + \gamma$ with a twofold ambiguity (for the moment, we put aside the ambiguity due to the sign of $\Delta\Gamma_s$).

The advantage of these decays is that there is a third decay which is related: $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0\phi$, where D_{CP}^0 is a CP eigenstate (either CP-odd or CP-even). In our analysis we consider D_{CP}^0 as the CP-even superposition $(D^0 + \bar{D}^0)/\sqrt{2}$. The amplitudes for the decays are then given by

$$\begin{aligned} \sqrt{2}A(B_s^0 \rightarrow D_{CP}^0\phi) &= -C_1^\phi e^{i\gamma} + C_2^\phi, \\ \sqrt{2}A(\bar{B}_s^0 \rightarrow D_{CP}^0\phi) &= C_1^\phi e^{-i\gamma} - C_2^\phi. \end{aligned} \quad (18)$$

By measuring the time-dependent decay amplitudes of $B_s^0(\bar{B}_s^0) \rightarrow D\phi$ ($D = D^0, \bar{D}^0, D_{CP}^0$), one can extract the magnitudes $|C_1^\phi|, |C_2^\phi|, |A_{D_{CP}}| = |A(B_s^0 \rightarrow D_{CP}^0\phi)|$ and $|\bar{A}_{D_{CP}}| = |A(\bar{B}_s^0 \rightarrow D_{CP}^0\phi)|$ (they are combinations of the overall normalizations and the C parameters [Eq. (10)]).

Using the first equation of Eq. (18), we define

$$\cos(\gamma + \delta_\phi) = \frac{2|A_{D_{CP}}|^2 - |C_1^\phi|^2 - |C_2^\phi|^2}{2|C_1^\phi||C_2^\phi|} \equiv \Sigma^+. \quad (19)$$

Similarly, from the second equation of Eq. (18), we get

$$\cos(\gamma - \delta_\phi) = \frac{2|\bar{A}_{D_{CP}}|^2 - |C_1^\phi|^2 - |C_2^\phi|^2}{2|C_1^\phi||C_2^\phi|} \equiv \Sigma^-. \quad (20)$$

Therefore, in the case of the $B_s^0(\bar{B}_s^0) \rightarrow D\phi$ decays, we have two more observables, Σ^+ and Σ^- . Combining Eqs. (17), (19) and (20), it is straightforward to find expressions for $\sin 2\beta_s, \cos 2\beta_s, \sin(2\beta_s + 2\gamma)$ and $\cos(2\beta_s + 2\gamma)$ in terms

of the above observables:

$$\begin{aligned}\sin 2\beta_s &= \frac{S_D^2 - \bar{S}_D^2 + \Sigma^{+2} - \Sigma^{-2}}{2(S_D \Sigma^+ - \bar{S}_D \Sigma^-)}, & \sin(2\beta_s + 2\gamma) &= \frac{S_D^2 - \bar{S}_D^2 - \Sigma^{+2} + \Sigma^{-2}}{2(S_D \Sigma^- - \bar{S}_D \Sigma^+)}, \\ \cos 2\beta_s &= \frac{S_D^2 - \bar{S}_D^2 - \Sigma^{+2} + \Sigma^{-2}}{2(\bar{A}_{\Delta\Gamma}^D \Sigma^- - A_{\Delta\Gamma}^D \Sigma^+)}, & \cos(2\beta_s + 2\gamma) &= \frac{S_D^2 - \bar{S}_D^2 + \Sigma^{+2} - \Sigma^{-2}}{2(\bar{A}_{\Delta\Gamma}^D \Sigma^+ - A_{\Delta\Gamma}^D \Sigma^-)},\end{aligned}\quad (21)$$

with

$$S_D^2 - \bar{S}_D^2 = -(A_{\Delta\Gamma}^D)^2 + (\bar{A}_{\Delta\Gamma}^D)^2. \quad (22)$$

Many years ago, $B_s^0(\bar{B}_s^0) \rightarrow D\phi$ decays were studied [31], but without the dependence on $\Delta\Gamma_s$. It was found that $\sin 2\beta_s$ and $\sin(2\beta_s + 2\gamma)$ could be obtained, which correspond to determining $2\beta_s$ with a twofold ambiguity and 2γ with a fourfold ambiguity. In the present case, the dependence on $\Delta\Gamma_s$ is included. This allows us to obtain $A_{\Delta\Gamma}^D$ and $\bar{A}_{\Delta\Gamma}^D$, which then permits us to measure $\cos 2\beta_s$ and $\cos(2\beta_s + 2\gamma)$, in addition to $\sin 2\beta_s$ and $\sin(2\beta_s + 2\gamma)$ [Eq. (21)]. These measurements allow an unambiguous determination of $2\beta_s$ and 2γ . We therefore see that a nonzero $\Delta\Gamma_s$ helps quite a bit in determining the weak phases. As has been discussed above, the sign of $\Delta\Gamma_s$ is not known, which implies that $A_{\Delta\Gamma}^D$ and $\bar{A}_{\Delta\Gamma}^D$ also have a sign ambiguity. This means that, in fact, $2\beta_s$ and γ are determined up to a twofold and fourfold³ ambiguity, respectively. Therefore, once we are able to fix the sign of $\Delta\Gamma_s$, the $B_s^0(\bar{B}_s^0) \rightarrow D\phi$ decays might be considered as an alternative mode to probe simultaneously γ and $2\beta_s$.

We therefore see that two-body $\bar{b} \rightarrow \bar{c}u\bar{s}/\bar{b} \rightarrow \bar{u}\bar{c}\bar{s}$ decays do not provide sufficient information to measure the CP phases $2\beta_s$ and 2γ in an unambiguous manner. In the next section we show that there are several ways to improve upon the two-body decay methods by using a Dalitz-plot analysis of the corresponding three-body decays.

III. THREE-BODY DECAYS

A. $B_s^0(\bar{B}_s^0) \rightarrow f, \bar{f}$

In recent years, it has been shown that one can get useful information from three-body B decays. For instance, time-integrated Dalitz-plot analyses of $B_s^0 \rightarrow K\pi\pi$ and $B_s^0 \rightarrow \pi K\bar{K}$ decays have been proposed as a probe of γ [34]. And various tests of the SM, as well as the extraction of weak phases, have been examined in the context of $B \rightarrow K\pi\pi$, $B \rightarrow K\bar{K}K$, $B \rightarrow \pi\bar{K}K$ and $B \rightarrow \pi\pi\pi$ decays [35].

In the previous section we discussed two-body $\bar{b} \rightarrow \bar{c}u\bar{s}/\bar{b} \rightarrow \bar{u}\bar{c}\bar{s}$ decays; in this section we examine the corresponding three-body decays. In $B_s^0(\bar{B}_s^0) \rightarrow PPP$ decays which receive a tree contribution, there are 5 final-state (f, \bar{f}) pairs: $(D_s^- K^+ \pi^0, D_s^+ K^- \pi^0)$, $(D_s^- K^0 \pi^+, D_s^+ \bar{K}^0 \pi^-)$, $(D^- K^+ \bar{K}^0, D^+ K^0 \bar{K}^-)$, $(\bar{D}^0 K^+ K^-, D^0 K^+ K^-)$, and $(\bar{D}^0 K^0 \bar{K}^0, D^0 K^0 \bar{K}^0)$. The CKM matrix elements of these decays are the same as in the corresponding two-body decay modes, and will therefore exhibit very similar time-dependent CP asymmetries.

The decay amplitude of $B_s^0(\bar{B}_s^0) \rightarrow PPP$ receives several different contributions, both resonant and non-resonant. In the following, we perform a time-dependent Dalitz-plot analysis of the three-body decays. This permits the measurement of each of the contributing amplitudes, as well as their relative phases. As we will see below, the Dalitz-plot analysis reduces the ambiguity in the measurement of γ and $2\beta_s$ compared to the corresponding two-body decays. We also show how this analysis resolves the sign ambiguity in $\Delta\Gamma_s$.

B. Dalitz-plot analysis

Here we review certain aspects of the Dalitz-plot analysis. We focus on the general three-body decay $B \rightarrow P_1 P_2 P_3$. We define the Dalitz-plot variables

$$s_{12} \equiv (p_1 + p_2)^2, \quad s_{13} \equiv (p_1 + p_3)^2, \quad s_{23} \equiv (p_2 + p_3)^2, \quad (23)$$

³ Using Eq. (21), we can determine $\cos 2\gamma$ without any ambiguity, whereas, due to the unknown sign of $A_{\Delta\Gamma}^D$ or $\bar{A}_{\Delta\Gamma}^D$, $\sin 2\gamma$ can be determined only with a twofold ambiguity. Combining these two results, 2γ can therefore be determined with a twofold ambiguity (or γ with a fourfold ambiguity).

which are related by the conservation law

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2 . \quad (24)$$

This shows that there are only two independent variables (below, we use s_{12} and s_{13}).

$B \rightarrow P_1 P_2 P_3$ can take place either via intermediate resonances or non-resonant contributions. A widely-used approximation in the parametrization of the decay amplitude is the isobar model. In this model, the individual terms are interpreted as complex production amplitudes for two-body resonances, and one also includes a term describing the non-resonant component. The amplitude is then written as

$$\mathcal{A}(s_{12}, s_{13}) = \sum_j a_j F_j(s_{12}, s_{13}) , \quad (25)$$

where the sum is over all decay modes (resonant and non-resonant). Here, the a_j are the complex coefficients describing the magnitudes and phases of different decay channels, while the $F_j(s_{12}, s_{13})$ contain the strong dynamics. The CP-conjugate amplitude is given by

$$\bar{\mathcal{A}}(s_{12}, s_{13}) = \sum_j \bar{a}_j \bar{F}_j(s_{13}, s_{12}) , \quad (26)$$

where $\bar{F}_j(s_{13}, s_{12}) = F_j(s_{12}, s_{13})$.

Now, in the experimental analysis, the $F_j(s_{12}, s_{13})$ take different (known) forms for the various contributions. By performing a maximum likelihood fit over the entire Dalitz plot, one can obtain the magnitudes and relative phases of the a_j , and similarly for the \bar{a}_j . Thus, the full decay amplitudes can be obtained.

C. $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$

In this subsection we focus specifically on the decay $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$, and use a modification of the method elaborated in Ref. [36]. The Dalitz-plot variables are

$$s^+ \equiv (p_{D_s} + p_\pi)^2 , \quad s^- \equiv (p_K + p_\pi)^2 , \quad s^0 \equiv (p_{D_s} + p_K)^2 . \quad (27)$$

The amplitudes are written as

$$\mathcal{A}(s^+, s^-) = \sum_j a_j F_j(s^+, s^-) , \quad \bar{\mathcal{A}}(s^+, s^-) = \sum_j \bar{a}_j \bar{F}_j(s^-, s^+) . \quad (28)$$

The time-dependent decay rates for the oscillating $B_s^0(t)$ and $\bar{B}_s^0(t)$ mesons, decaying to the same final state f , are given by

$$\begin{aligned} \Gamma(B_s^0(t) \rightarrow f) &\sim \frac{1}{2} e^{-\Gamma_s t} \left[A_{ch}(s^+, s^-) \cosh(\Delta\Gamma_s t/2) - A_{sh}(s^+, s^-) \sinh(\Delta\Gamma_s t/2) \right. \\ &\quad \left. + A_c(s^+, s^-) \cos(\Delta m_s t) - A_s(s^+, s^-) \sin(\Delta m_s t) \right] , \\ \Gamma(\bar{B}_s^0(t) \rightarrow f) &\sim \frac{1}{2} e^{-\Gamma_s t} \left[A_{ch}(s^-, s^+) \cosh(\Delta\Gamma_s t/2) - A_{sh}(s^-, s^+) \sinh(\Delta\Gamma_s t/2) \right. \\ &\quad \left. - A_c(s^-, s^+) \cos(\Delta m_s t) + A_s(s^-, s^+) \sin(\Delta m_s t) \right] . \end{aligned} \quad (29)$$

Here

$$\begin{aligned} A_{ch}(s^+, s^-) &= |\mathcal{A}(s^+, s^-)|^2 + |\bar{\mathcal{A}}(s^+, s^-)|^2 , \\ A_c(s^+, s^-) &= |\mathcal{A}(s^+, s^-)|^2 - |\bar{\mathcal{A}}(s^+, s^-)|^2 , \\ A_{sh}(s^+, s^-) &= 2\text{Re}(e^{-2i\beta_s} \bar{\mathcal{A}}(s^+, s^-) \mathcal{A}^*(s^+, s^-)) , \\ A_s(s^+, s^-) &= 2\text{Im}(e^{-2i\beta_s} \bar{\mathcal{A}}(s^+, s^-) \mathcal{A}^*(s^+, s^-)) , \end{aligned} \quad (30)$$

where $\mathcal{A}(s^+, s^-)$ and $\bar{\mathcal{A}}(s^+, s^-)$ are given in Eq. (28).

Now, there are a number of resonances which contribute to $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$. For illustrative purposes, we consider just two of them: $D_s^\pm K^{*\mp}(892)$ and $D_s^{*\pm} K^\mp$. The decays $K^{*\pm} \rightarrow K^\pm \pi^0$ and $D_s^{*\pm} \rightarrow D_s^\pm \pi^0$ have already

been observed: $B(K^{*\pm}(892) \rightarrow K^\pm \pi^0) = 50\%$, $B(D_s^{*\pm} \rightarrow D_s^\pm \pi^0) = (5.8 \pm 0.7)\%$ [37]. $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^{*\mp}$ and $B_s^0(\bar{B}_s^0) \rightarrow D_s^{*\pm} K^\mp$ are the $B_s^0(\bar{B}_s^0) \rightarrow VP$ equivalents of the decay discussed in Sec. II, $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp$. The additional ingredient here is that we also consider the decay products of the V , so that we have the full decay chain $B_s^0(\bar{B}_s^0) \rightarrow VP \rightarrow PPP$.

For these two resonances, we have

$$\begin{aligned}\mathcal{A}_{K^*}(B_s^0 \rightarrow D_s^- K^{*+} \rightarrow D_s^- K^+ \pi^0) &= a_1^{K^*} e^{i\gamma} F_{K^*}, \\ \bar{\mathcal{A}}_{K^*}(\bar{B}_s^0 \rightarrow D_s^- K^{*+} \rightarrow D_s^- K^+ \pi^0) &= a_2^{K^*} F_{K^*}, \\ \mathcal{A}_{D_s^*}(B_s^0 \rightarrow D_s^{*-} K^+ \rightarrow D_s^- K^+ \pi^0) &= a_1^{D_s^*} e^{i\gamma} F_{D_s^*}, \\ \bar{\mathcal{A}}_{D_s^*}(\bar{B}_s^0 \rightarrow D_s^{*-} K^+ \rightarrow D_s^- K^+ \pi^0) &= a_2^{D_s^*} F_{D_s^*}.\end{aligned}\tag{31}$$

Including both resonances, the amplitudes of $B_s^0(\bar{B}_s^0) \rightarrow D_s^- K^+ \pi^0$ are

$$\begin{aligned}\mathcal{A} &= e^{i\gamma}(a_1^{K^*} F_{K^*} + a_1^{D_s^*} F_{D_s^*}), \\ \bar{\mathcal{A}} &= (a_2^{K^*} F_{K^*} + a_2^{D_s^*} F_{D_s^*}).\end{aligned}\tag{32}$$

With these amplitudes, A_{ch} , A_c , A_{sh} and A_s [Eq. (30)] take the forms

$$\begin{aligned}A_{ch}^{D_s K \pi} &= \left(|a_1^{K^*}|^2 + |a_2^{K^*}|^2\right) |F_{K^*}|^2 + \left(|a_1^{D_s^*}|^2 + |a_2^{D_s^*}|^2\right) |F_{D_s^*}|^2 \\ &\quad + 2\text{Re}\left((a_1^{K^*} F_{K^*})^* (a_1^{D_s^*} F_{D_s^*})\right) + 2\text{Re}\left((a_2^{K^*} F_{K^*})^* (a_2^{D_s^*} F_{D_s^*})\right), \\ A_c^{D_s K \pi} &= \left(|a_1^{K^*}|^2 - |a_2^{K^*}|^2\right) |F_{K^*}|^2 + \left(|a_1^{D_s^*}|^2 - |a_2^{D_s^*}|^2\right) |F_{D_s^*}|^2 \\ &\quad + 2\text{Re}\left((a_1^{K^*} F_{K^*})^* (a_1^{D_s^*} F_{D_s^*})\right) - 2\text{Re}\left((a_2^{K^*} F_{K^*})^* (a_2^{D_s^*} F_{D_s^*})\right), \\ A_{sh}^{D_s K \pi} &= \cos(2\beta_s + \gamma + \delta_{K^*}) |a_1^{K^*}| |a_2^{K^*}| |F_{K^*}|^2 + \cos(2\beta_s + \gamma + \delta_{D_s^*}) |a_1^{D_s^*}| |a_2^{D_s^*}| |F_{D_s^*}|^2 \\ &\quad + \cos(2\beta_s + \gamma + \delta) |a_1^{K^*}| |a_2^{D_s^*}| \text{Re}(F_{K^*}^* F_{D_s^*}) \\ &\quad - \sin(2\beta_s + \gamma + \delta) |a_1^{K^*}| |a_2^{D_s^*}| \text{Im}(F_{K^*}^* F_{D_s^*}) \\ &\quad + \text{Re}\left[e^{-i(2\beta_s + \gamma)} (a_1^{D_s^*} F_{D_s^*})^* (a_2^{K^*} F_{K^*})\right], \\ A_s^{D_s K \pi} &= -\sin(2\beta_s + \gamma + \delta_{K^*}) |a_1^{K^*}| |a_2^{K^*}| |F_{K^*}|^2 - \sin(2\beta_s + \gamma + \delta_{D_s^*}) |a_1^{D_s^*}| |a_2^{D_s^*}| |F_{D_s^*}|^2 \\ &\quad - \sin(2\beta_s + \gamma + \delta) |a_1^{K^*}| |a_2^{D_s^*}| \text{Re}(F_{K^*}^* F_{D_s^*}) \\ &\quad + \cos(2\beta_s + \gamma + \delta) |a_1^{K^*}| |a_2^{D_s^*}| \text{Im}(F_{K^*}^* F_{D_s^*}) \\ &\quad + \text{Im}\left[e^{-i(2\beta_s + \gamma)} (a_1^{D_s^*} F_{D_s^*})^* (a_2^{K^*} F_{K^*})\right],\end{aligned}\tag{33}$$

with $\delta_{K^*} = -\arg((a_1^{K^*})^* a_2^{K^*})$, $\delta_{D_s^*} = -\arg((a_1^{D_s^*})^* a_2^{D_s^*})$, and $\delta = -\arg((a_1^{K^*})^* a_2^{D_s^*})$.

Above, in the discussion of the time-independent Dalitz-plot analysis, we noted that the magnitudes and relative phases of the a_j can be obtained from a maximum likelihood fit over the entire Dalitz plot, given assumed forms for the F_j 's. The same holds true for the time-dependent Dalitz-plot analysis – the magnitudes and relative phases of the contributing resonances, i.e. $a_1^{K^*}$, $a_2^{K^*}$, $a_1^{D_s^*}$, and $a_2^{D_s^*}$, can all be obtained. Indeed, such an analysis has been performed by the Babar and Belle collaborations for the decay $B_d^0(t) \rightarrow K_S \pi^+ \pi^-$ [38]. In particular, all the coefficients that multiply the $F_i^* F_j$ [Eq. (33)] bilinears can be obtained from a maximum likelihood fit to the corresponding Dalitz plot PDFs.

This permits the extraction of the weak phases. For example, we can extract $2\beta_s + \gamma + \delta$ without any ambiguity from the third and fourth terms of $A_s^{D_s K \pi}$. In a similar manner, the time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$ allows the extraction of the phase $2\beta_s + \gamma - \delta$. The combination of these two results yields $2\beta_s + \gamma$ and δ with a twofold ambiguity. And if factorization is imposed, the discrete ambiguity is removed entirely (only the solution with $\delta \simeq 0$ is kept). The key point here is that we do not use $A_{sh}^{D_s K \pi}$ at all. As a consequence, there is no discrete ambiguity due to the sign ambiguity of $\Delta\Gamma_s$ [see the discussion following Eq. (30)]. This is to be contrasted with two-body decays. There $2\beta_s + \gamma$ can also be obtained with a twofold ambiguity. However, because $\mathcal{A}_{\Delta\Gamma}$ and $\bar{\mathcal{A}}_{\Delta\Gamma}$ are used [Eq. (15)], there is an additional discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$.

We note that one can extract different trigonometric functions such as $|\sin(2\beta_s + \gamma + \delta)|$, $|\cos(2\beta_s + \gamma + \delta)|$, $|\cos(2\beta_s + \gamma + \delta_i)|$, etc., from $A_{sh}^{D_s K \pi}$ [Eq. (33)]. Due to the sign ambiguity of $\Delta\Gamma_s$, which can be viewed as the sign ambiguity in $A_{sh}^{D_s K \pi}$, the sign of these trigonometric functions cannot be determined. Depending on the sign of $\Delta\Gamma_s$, their sign could be positive or negative. Therefore, we can determine the sign of $\Delta\Gamma_s$ if we are able to fix the sign of these trigonometric functions. Now, the functions $\sin(2\beta_s + \gamma + \delta)$ and $\cos(2\beta_s + \gamma + \delta)$ can be extracted without ambiguity from $A_{sh}^{D_s K \pi}$, which fixes the sign of $\Delta\Gamma_s$ and hence removes the discrete ambiguity in $A_{sh}^{D_s K \pi}$. Note that this can be done without measuring ϕ_s . This method can therefore be used to determine the sign of $\cos\phi_s$.

In the above, we have concentrated on the decay $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$. However, any of the decay pairs discussed in Sec. III can be used. All that is necessary is that there be at least two resonances contributing to the decay. We therefore see that, by using such three-body decays, one can obtain $2\beta_s + \gamma$ (with a twofold ambiguity if factorization is not assumed), as well as resolve the sign ambiguity in $\Delta\Gamma_s$. The resolution of the $\Delta\Gamma_s$ sign ambiguity determines the sign of $\cos\phi_s$. The precise knowledge of γ from other measurements allows one to obtain $2\beta_s$ with a twofold ambiguity (since $2\beta_s + \gamma$ can itself be extracted with a twofold ambiguity), which can be compared with the measurement of $2\beta_s$ from $B_s^0 \rightarrow J/\psi\phi$ [Eq. (7)].

Still, it is preferable to have a method that allows the direct determination of $2\beta_s$ and γ individually. This can be done by measuring the decay $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$, which is discussed in the next subsection.

D. $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$

In Sec. II C we discussed the two-body decays $B_s^0(\bar{B}_s^0) \rightarrow D\phi$ ($D = D^0, \bar{D}^0, D_{CP}^0$), and showed that it is possible to extract $2\beta_s$ and 2γ with a twofold ambiguity due to the unknown sign of $\Delta\Gamma_s$. The time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \rightarrow D^0 K \bar{K}, \bar{D}^0 K \bar{K}$ is similar to that of the previous subsection, with the intermediate resonances $\phi(1020)$ or $f_0(1500)$ decaying to the final state $K \bar{K}$. In this subsection we consider in addition the related three-body decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$, with $D_{CP}^0 \equiv 1/\sqrt{2}(D^0 \pm \bar{D}^0)$.

$B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$ receives contributions from several different intermediate resonances: $\phi(1020)$, $\phi(1680)$, $f_0(1500)$, $f_0(1710)$, $D_{s_j}^{*\pm}$, etc., which follow the decay chains $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 \phi \rightarrow D_{CP}^0 K^+ K^-$, $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 f_0 \rightarrow D_{CP}^0 K^+ K^-$, $B_s^0(\bar{B}_s^0) \rightarrow D_{s_j}^{*\pm} K^\mp \rightarrow D_{CP}^0 K^\pm K^\mp$. To simplify our analysis, we consider only the $\phi(1020)$ and $f_0(1500)$ resonances. The amplitude with an intermediate ϕ resonance can be written as

$$\begin{aligned}\sqrt{2}A_\phi(B_s^0 \rightarrow D_{CP}^0 \phi(\rightarrow K^+ K^-)) &= A(B_s^0 \rightarrow D^0 K^+ K^-) + A(B_s^0 \rightarrow \bar{D}^0 K^+ K^-), \\ \sqrt{2}\bar{A}_\phi(\bar{B}_s^0 \rightarrow D_{CP}^0 \phi(\rightarrow K^+ K^-)) &= A(\bar{B}_s^0 \rightarrow D^0 K^+ K^-) + A(\bar{B}_s^0 \rightarrow \bar{D}^0 K^+ K^-),\end{aligned}\quad (34)$$

where

$$\begin{aligned}A(B_s^0 \rightarrow D^0 \phi \rightarrow D^0 K^+ K^-) &= -C_1^\phi e^{i\gamma} F_\phi, \\ A(\bar{B}_s^0 \rightarrow D^0 \phi \rightarrow D^0 K^+ K^-) &= -C_2^\phi F_\phi, \\ A(B_s^0 \rightarrow \bar{D}^0 \phi \rightarrow D^0 K^+ K^-) &= C_2^\phi F_\phi, \\ A(\bar{B}_s^0 \rightarrow \bar{D}^0 \phi \rightarrow D^0 K^+ K^-) &= C_1^\phi e^{-i\gamma} F_\phi.\end{aligned}\quad (35)$$

The amplitude with an intermediate f_0 resonance is given by a similar expression, with the replacement $\phi \rightarrow f_0$. Including the contributions from these two resonances, the total amplitude can be written as

$$\begin{aligned}A(B_s^0 \rightarrow D_{CP}^0 K^+ K^-) &= A_\phi(B_s^0 \rightarrow D_{CP}^0 K^+ K^-) + A_{f_0}(B_s^0 \rightarrow D_{CP}^0 K^+ K^-), \\ \bar{A}(\bar{B}_s^0 \rightarrow D_{CP}^0 K^+ K^-) &= \bar{A}_\phi(\bar{B}_s^0 \rightarrow D_{CP}^0 K^+ K^-) + \bar{A}_{f_0}(\bar{B}_s^0 \rightarrow D_{CP}^0 K^+ K^-).\end{aligned}\quad (36)$$

With these, A_c^{DKK} , A_{ch}^{DKK} and A_s^{DKK} can be computed similarly to Eq. (33). First, we have

$$\begin{aligned}A_c^{DKK} &= \sum_{i=\phi, f_0} [(|A_i|^2 - |\bar{A}_i|^2) + 2\text{Re}(A_\phi A_{f_0}^* - \bar{A}_\phi \bar{A}_{f_0}^*)], \\ A_{ch}^{DKK} &= \sum_{i=\phi, f_0} [(|A_i|^2 + |\bar{A}_i|^2) + 2\text{Re}(A_\phi A_{f_0}^* + \bar{A}_\phi \bar{A}_{f_0}^*)],\end{aligned}\quad (37)$$

in which

$$\begin{aligned}
\text{Re}(A_\phi A_{f_0}^* - \bar{A}_\phi \bar{A}_{f_0}^*) &= |C_2^{f_0}| |C_2^\phi| \sin \gamma \left[r_\phi \{ \sin \delta_\phi \text{Re}(F_\phi F_{f_0}^*) + \cos \delta_\phi \text{Im}(F_\phi F_{f_0}^*) \} \right. \\
&\quad \left. + r_{f_0} \{ \sin \delta_{f_0} \text{Re}(F_\phi F_{f_0}^*) - \cos \delta_{f_0} \text{Im}(F_\phi F_{f_0}^*) \} \right], \\
\text{Re}(A_\phi A_{f_0}^* + \bar{A}_\phi \bar{A}_{f_0}^*) &= |C_2^{f_0}| |C_2^\phi| \left[\text{Re}(F_\phi F_{f_0}^*) - r_\phi \cos \gamma \{ \cos \delta_\phi \text{Re}(F_\phi F_{f_0}^*) - \sin \delta_\phi \text{Im}(F_\phi F_{f_0}^*) \} \right. \\
&\quad \left. - r_{f_0} \cos \gamma \{ \cos \delta_{f_0} \text{Re}(F_\phi F_{f_0}^*) + \sin \delta_{f_0} \text{Im}(F_\phi F_{f_0}^*) \} \right. \\
&\quad \left. + r_\phi r_{f_0} \{ \cos(\delta_\phi - \delta_{f_0}) \text{Re}(F_\phi F_{f_0}^*) - \sin(\delta_\phi - \delta_{f_0}) \text{Im}(F_\phi F_{f_0}^*) \} \right], \quad (38)
\end{aligned}$$

where $r_i = |C_1^i|/|C_2^i|$ and $\delta_i = \arg(C_1^i/C_2^i)$ ($i = \phi, f_0$). Using Eq. (38) in Eq. (29), a maximum likelihood fit to the Dalitz-plot PDFs allows one to extract

$$\begin{aligned}
|C_2^{f_0}| |C_2^\phi| r_i \cos \gamma \cos \delta_i &\equiv \sigma_c^i, \\
|C_2^{f_0}| |C_2^\phi| r_i \sin \gamma \cos \delta_i &\equiv \sigma_s^i. \quad (39)
\end{aligned}$$

This gives the ratio

$$\frac{\sigma_s^i}{\sigma_c^i} = \tan \gamma. \quad (40)$$

Since the hadronic uncertainties cancel in the ratio, it yields a theoretically clean determination of the angle γ with a twofold ambiguity, even without the knowledge of the strong phases.

Second, we have

$$A_s^{DKK} = \text{Im} [e^{-2i\beta_s} \mathcal{A}^* \bar{\mathcal{A}}] = \text{Im} [e^{-2i\beta_s} (A_\phi^* \bar{A}_\phi + A_\phi^* \bar{A}_{f_0} + A_{f_0}^* \bar{A}_\phi + A_{f_0}^* \bar{A}_{f_0})] . \quad (41)$$

The first and fourth terms of A_s^{DKK} are given by

$$\text{Im} [e^{-2i\beta_s} A_i^* \bar{A}_i] = \frac{1}{2} \text{Im} [e^{-2i\beta_s} |C_2^i|^2 |F_i|^2 (1 + r_i^2 e^{-2i\gamma} + r_i (e^{i(\delta_\phi - \gamma)} + e^{-i(\delta_\phi + \gamma)}))] , \quad (42)$$

which allows the extraction of $\sin 2\beta_s$, $\sin(2\beta_s + 2\gamma)$, $\sin(2\beta_s + \gamma - \delta_{\phi/f_0})$ and $\sin(2\beta_s + \gamma + \delta_{\phi/f_0})$. The ϕ - f_0 interference terms are given by

$$\begin{aligned}
\text{Im} [e^{-2i\beta_s} (A_\phi^* \bar{A}_{f_0} + A_{f_0}^* \bar{A}_\phi)] &= \frac{1}{2} \text{Im} \left[e^{-2i\beta_s} |C_2^\phi| |C_2^{f_0}| \left\{ - (F_\phi^* F_{f_0} + F_{f_0}^* F_\phi) \right. \right. \\
&\quad \left. + r_\phi \left(e^{-i(\gamma + \delta_\phi)} F_\phi^* F_{f_0} + e^{-i(\gamma - \delta_\phi)} F_{f_0}^* F_\phi \right) \right. \\
&\quad \left. + r_{f_0} \left(e^{-i(\gamma - \delta_{f_0})} F_\phi^* F_{f_0} + e^{-i(\gamma + \delta_{f_0})} F_{f_0}^* F_\phi \right) \right. \\
&\quad \left. \left. - r_\phi r_{f_0} \left(e^{-i(2\gamma + \delta_\phi - \delta_{f_0})} F_\phi^* F_{f_0} + e^{-i(2\gamma - \delta_\phi + \delta_{f_0})} F_{f_0}^* F_\phi \right) \right\} \right] . \quad (43)
\end{aligned}$$

This yields

$$\begin{aligned}
\text{Im} [e^{-2i\beta_s} (A_\phi^* \bar{A}_{f_0} + A_{f_0}^* \bar{A}_\phi)] &= \frac{1}{2} |C_2^\phi| |C_2^{f_0}| \left[\sin 2\beta_s \{ \text{Re}(F_\phi^* F_{f_0}) + \text{Re}(F_{f_0}^* F_\phi) \} \right. \\
&\quad - r_\phi \{ \sin(2\beta_s + \gamma + \delta_\phi) \text{Re}(F_\phi^* F_{f_0}) - \cos(2\beta_s + \gamma + \delta_\phi) \text{Im}(F_\phi^* F_{f_0}) \\
&\quad + \sin(2\beta_s + \gamma - \delta_\phi) \text{Re}(F_{f_0}^* F_\phi) - \cos(2\beta_s + \gamma - \delta_\phi) \text{Im}(F_{f_0}^* F_\phi) \} \\
&\quad - r_{f_0} \{ \sin(2\beta_s + \gamma + \delta_{f_0}) \text{Re}(F_{f_0}^* F_\phi) - \cos(2\beta_s + \gamma + \delta_{f_0}) \text{Im}(F_{f_0}^* F_\phi) \\
&\quad + \sin(2\beta_s + \gamma - \delta_{f_0}) \text{Re}(F_\phi^* F_{f_0}) - \cos(2\beta_s + \gamma - \delta_{f_0}) \text{Im}(F_\phi^* F_{f_0}) \} \\
&\quad + r_\phi r_{f_0} \{ \sin(\delta_\phi - \delta_{f_0} + 2\beta_s + 2\gamma) \text{Re}(F_\phi^* F_{f_0}) - \cos(\delta_\phi - \delta_{f_0} + 2\beta_s + 2\gamma) \text{Im}(F_\phi^* F_{f_0}) \\
&\quad + \sin(\delta_{f_0} - \delta_\phi + 2\beta_s + 2\gamma) \text{Re}(F_{f_0}^* F_\phi) - \cos(\delta_{f_0} - \delta_\phi + 2\beta_s + 2\gamma) \text{Im}(F_{f_0}^* F_\phi) \} \left. \right] . \quad (44)
\end{aligned}$$

From the above, we can extract

$$\begin{aligned} -r_i \sin(2\beta_s + \gamma \pm \delta_i) &\equiv S_{DKK}^{i\pm}, & r_i \cos(2\beta_s + \gamma \pm \delta_i) &\equiv C_{DKK}^{i\pm}, & \sin 2\beta_s &\equiv S_{DKK}, \\ r_{ij} \sin(2\beta_s + 2\gamma \pm \delta_{ij}) &\equiv S_{DKK}^{ij\pm}, & -r_{ij} \cos(2\beta_s + 2\gamma \pm \delta_{ij}) &\equiv C_{DKK}^{ij\pm}, \end{aligned} \quad (45)$$

where $r_{ij} \equiv r_i r_j$ and the corresponding $\delta_{ij} \equiv \delta_i - \delta_j$ ($i, j = \phi, f_0$). It is straightforward to find expressions for $\tan(2\beta_s + \gamma)$ and $\tan(2\beta_s + 2\gamma)$ in terms of the above observables:

$$\tan(2\beta_s + \gamma) = -\frac{S_{DKK}^{i+} + S_{DKK}^{i-}}{C_{DKK}^{i+} + C_{DKK}^{i-}}, \quad \tan(2\beta_s + 2\gamma) = -\frac{S_{DKK}^{ij+} + S_{DKK}^{ij-}}{C_{DKK}^{ij+} + C_{DKK}^{ij-}}. \quad (46)$$

With these, one can obtain the expression for $\tan \gamma$ in terms of the extracted observables:

$$\tan \gamma = \frac{\tan(2\beta_s + 2\gamma) - \tan(2\beta_s + \gamma)}{1 - \tan(2\beta_s + \gamma) \tan(2\beta_s + 2\gamma)}. \quad (47)$$

This way of getting $\tan \gamma$ uses A_s^{DKK} [see also Eq. (40)].

Combining Eqs. (46) and (47), we obtain

$$\tan 2\beta_s = \frac{\tan(2\beta_s + \gamma) - \tan \gamma}{1 - \tan(2\beta_s + \gamma) \tan \gamma}. \quad (48)$$

This determines $2\beta_s$ with the twofold ambiguity $2\beta_s \rightarrow \pi + 2\beta_s$. However, as we note in Eq. (45), we can extract $\sin 2\beta_s$ without any sign ambiguity. This determines $2\beta_s$ with the twofold ambiguity $2\beta_s \rightarrow \pi - 2\beta_s$, which is different from that obtained in $\tan 2\beta_s$. Therefore, the combined measurements of $\tan 2\beta_s$ and $\sin 2\beta_s$ allow us to extract $2\beta_s$ without any ambiguity. The sign ambiguity in $\Delta\Gamma_s$ can be resolved in a similar way to that discussed in Sec. III C.

Above, we discussed the interference between the two resonance states $\phi(1020)$ and $f_0(1500)$. However, the analysis would hold equally for the interference between any two resonances decaying to the same final state. Similar information can also be obtained from the time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K^0 \bar{K}^0$.

IV. EXTRACTION OF ϕ_s AND $|\Gamma_{12}^s|$

In the previous section(s) we examined methods for extracting the CP phase $2\beta_s$ using various two- and three-body decays. The idea is that if a nonzero value of $2\beta_s$ is found, this will be clear evidence of new physics in B_s^0 - \bar{B}_s^0 mixing. In addition, if such a value of $2\beta_s$ is obtained, we will want to know its exact value in order to ascertain which different models of NP could generate such mixing. To this end, the best method will be that for which the discrete ambiguity in $2\beta_s$ is minimized. However, there is one question which has not yet been addressed: if NP in the mixing is found, does it contribute to Γ_{12}^s in addition to M_{12}^s ?

In order to answer this question, ϕ_s and $|\Gamma_{12}^s|$ must be measured and their values compared with those predicted by the SM [Eq. (6)]. As pointed out by several authors, the possibility of NP in Γ_{12}^s cannot be ruled out, and NP can enhance Γ_{12}^s significantly above the SM prediction, even including the error bars. In order to measure or extract both ϕ_s and $|\Gamma_{12}^s|$, we need (at least) two observables which are sensitive to NP effects in Γ_{12}^s . We choose $\Delta\Gamma_s$ and the semileptonic asymmetry a_{sl}^s . As we show below, the precise measurement of these two observables can be used to extract ϕ_s and $|\Gamma_{12}^s|$.

The expression for $\Delta\Gamma_s$ is given in Eq. (4); the semileptonic asymmetry is defined as

$$a_{sl}^s = \text{Im} \left[\frac{\Gamma_{12}^s}{M_{12}^s} \right] = \frac{2|\Gamma_{12}^s|}{\Delta M_s} \sin \phi_s. \quad (49)$$

Combining Eqs. (4) and (49) we obtain

$$\begin{aligned} \tan \phi_s &= \frac{a_{sl}^s \Delta M_s}{\Delta\Gamma_s}, \\ |\Gamma_{12}^s| &= \frac{\sqrt{\Delta\Gamma_s^2 + a_{sl}^s{}^2 \Delta M_s^2}}{2}. \end{aligned} \quad (50)$$

Now, ΔM_s is known very precisely – $\Delta M_s = 17.77 \pm 0.12$ [8, 39] – and so the precise measurements of a_{sl}^s and $\Delta\Gamma_s$ (without sign ambiguity) allow one to extract ϕ_s without any ambiguity⁴. This then determines $|\Gamma_{12}^s|$.

CDF and DØ have measured a_{sl}^s directly and the average of their measurements is given by [39]

$$a_{sl}^s = -0.0115 \pm 0.0061 . \quad (51)$$

If we take $|\Delta\Gamma_s| = 0.075 \pm 0.04$, as given by CDF [20], we obtain

$$\tan \phi_s = -2.72 \pm 2.05 , \quad |\Gamma_{12}^s| = 0.11 \pm 0.05 \text{ ps}^{-1} , \quad (52)$$

while $|\Delta\Gamma_s| = 0.163_{-0.064}^{+0.065}$, as given by DØ [21], yields

$$\tan \phi_s = -1.25 \pm 0.83 , \quad |\Gamma_{12}^s| = 0.13 \pm 0.05 \text{ ps}^{-1} . \quad (53)$$

Although $|\Gamma_{12}^s|$ and ϕ_s can significantly deviate from their SM predictions [Eq. (6)], both of them are consistent with the SM within the error bar. Note that in the above numerical analysis we do not consider the negative solution for $\Delta\Gamma_s$, which introduces a sign ambiguity in the extraction of ϕ_s . It is clear that improved measurements of both a_{sl}^s and $\Delta\Gamma_s$ are essential in order to understand the underlying physics of B_s^0 - \bar{B}_s^0 mixing and the width difference.

In this paper we have focused on methods for measuring $\Delta\Gamma_s$ using three-body decays. In practice, this will be carried out as follows. For definitiveness, consider the decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K^+ K^-$. Generalising Eq. (29) to $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K^+ K^-$, the time-dependent untagged differential decay distribution is given by

$$\begin{aligned} \Gamma_{\text{untagged}}(D_{CP}^0 K^+ K^-, t) &= \frac{d^2\Gamma(B_s^0 \rightarrow D_{CP}^0 K^+ K^-)}{ds^+ ds^-} + \frac{d^2\Gamma(\bar{B}_s^0 \rightarrow D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\equiv e^{-\Gamma_s t} [A_{ch}^{DKK} \cosh(\Delta\Gamma_s t/2) + A_{sh}^{DKK} \sinh(\Delta\Gamma_s t/2)] , \end{aligned} \quad (54)$$

where A_{ch}^{DKK} and A_{sh}^{DKK} are defined in Eq. (30). Neglecting terms of order $(\Delta\Gamma_s/\Gamma_s^2)^2$ and higher, the time-integrated differential untagged decay distribution is given by

$$\int_0^\infty dt \Gamma_{\text{untagged}}(D_{CP}^0 K^+ K^-, t) = \frac{1}{4\Gamma_s} \left[A_{ch}^{DKK} + 2A_{sh}^{DKK} \frac{\Delta\Gamma_s}{\Gamma_s} \right] . \quad (55)$$

For a single resonance, say ϕ ,

$$\begin{aligned} A_{ch}^{DKK} &= A_\phi^2 + \bar{A}_\phi^2, \\ A_{sh}^{DKK} &= \text{Re} \left[e^{-2i\beta_s} |C_2^\phi|^2 |F_\phi|^2 \{ 1 + r_\phi^2 e^{-2i\gamma} + r_\phi (e^{-i(\gamma+\delta_\phi)} + e^{-i(\gamma-\delta_\phi)}) \} \right] . \end{aligned} \quad (56)$$

As discussed in the previous section, A_{ch}^{DKK} is fully known from the CP-averaged branching fraction of the intermediate resonance ϕ . Once we have enough precision, a fit to the distribution given by Eq. (54) or (55) allows one to obtain $\Delta\Gamma_s$ and the various coefficients of $|F_\phi|^2$ (which yields $2\beta_s$). Such a fit will not allow the determination of the sign of $\Delta\Gamma_s$ or $\cos \phi_s$, but Eq. (50) can still be used to obtain ϕ_s (with a twofold ambiguity) and $|\Gamma_{12}^s|$.

However, the above fit, though possible, is made difficult due to the requirement of having to simultaneously extract $\Delta\Gamma_s$ and the components of A_{ch}^{DKK} . Given this, we would rather propose an alternative procedure. Referring again to Eq. (29), the time-dependent tagged differential decay distribution is given by

$$\begin{aligned} \Gamma_{\text{tagged}}(D_{CP}^0 K^+ K^-, t) &= \frac{d^2\Gamma(B_s^0 \rightarrow D_{CP}^0 K^+ K^-)}{ds^+ ds^-} - \frac{d^2\Gamma(\bar{B}_s^0 \rightarrow D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\equiv e^{-\Gamma_s t} [A_c^{DKK} \cos(\Delta m_s t/2) - A_s^{DKK} \sin(\Delta m_s t/2)] , \end{aligned} \quad (57)$$

where A_c^{DKK} and A_s^{DKK} are defined in Eqs. (30), (37) and (44). From a fit to the above distribution, one can extract only the coefficients of different bilinears in A_s^{DKK} and A_c^{DKK} , since Δm_s is known. Thus, this fit straightforwardly gives information regarding A_c^{DKK} and A_s^{DKK} . As discussed in the previous section, from A_s^{DKK} alone we can extract $2\beta_s$ and $\gamma + 2\beta_s \pm \delta_\phi$ without any ambiguity, and γ with the ambiguity $[\gamma, \pi + \gamma]$. This permits the reconstruction of A_{sh}^{DKK} [Eq. (56)]. That is, all the coefficients of $|F_\phi|^2$ in A_{sh}^{DKK} can be obtained from a fit to Eq. (57). With this

⁴ Knowledge of $\tan \phi_s$ gives ϕ_s with a twofold ambiguity, $\phi_s \leftrightarrow \pi + \phi_s$. However, a_{sl}^s determines $\sin \phi_s$, which allows one to differentiate ϕ_s and $\pi + \phi_s$.

knowledge, there is only one unknown in Eq. (54) or (55) – $\Delta\Gamma_s$ – and this can be determined by a fit. This may be a somewhat simpler procedure. Once we are able to measure $\Delta\Gamma_s$, then, along with a_{sl}^s and ΔM_s , Eq. (50) can be used to obtain the CP phase ϕ_s and $|\Gamma_{12}^s|$.

The above analysis is also applicable to the decays $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$. However, as was discussed in Sec. III C, for such decays all the trigonometric functions in $A_{sh}^{D_s K \pi}$ are not fully known – the only known functions are those appearing as the coefficients of $\text{Im}(F_i F_j^*)$ or $\text{Re}(F_i F_j^*)$ ($i \neq j$). Therefore, for such decays we can use Eq. (54) to fit $\Delta\Gamma_s$, but we need at least two interfering resonances, and only the terms proportional to $\text{Im}(F_i F_j^*)$ or $\text{Re}(F_i F_j^*)$ are useful [see $A_{ch}^{D_s K \pi}$ in Eq. (33)].

V. CONCLUSIONS

It is well known that the weak phase of B_s^0 - \bar{B}_s^0 mixing is very small in the SM: $2\beta_s \simeq 0$. If this quantity is measured to be significantly different from zero, this is a smoking-gun signal of new physics (NP). However, in general we would like more information from such a measurement. For instance, in order to distinguish among potential NP models, it is important to have an unambiguous determination of $2\beta_s$. Similarly, although the width difference $\Delta\Gamma_s$ between the two B_s mass eigenstates is positive in the SM, it can take either sign in the presence of NP. Ideally, a method probing B_s^0 - \bar{B}_s^0 mixing which relies on a nonzero $\Delta\Gamma_s$ should be able to remove its sign ambiguity. Finally, although it is usually assumed that NP contributes only to M_{12}^s , it has been shown that NP contributions to Γ_{12}^s can also be important. In order to explore this possibility, it is necessary to measure the CP phase ϕ_s and $|\Gamma_{12}^s|$.

In this paper, we examine a variety of methods of measuring B_s^0 - \bar{B}_s^0 mixing with an eye to addressing the above issues. We look at two- and three-body B_s decays with $\bar{b} \rightarrow \bar{c}u\bar{s}$ and $\bar{b} \rightarrow \bar{u}c\bar{s}$ transitions, concentrating on those final states which are accessible to both B_s^0 and \bar{B}_s^0 mesons (so that there is indirect CP violation). The time-dependent decay rates include both $\Delta m_s t$ and $\Delta\Gamma_s t$ terms.

We begin with a review of $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp$ decays. Considering sizeable $\Delta\Gamma_s$, we find that this method allows the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity. We then turn to $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 \phi$ decays, where D_{CP}^0 is a CP eigenstate. Here we find that $2\beta_s$ and 2γ can each be determined up to a twofold ambiguity. Here, the ambiguity is due to the unknown sign of $\Delta\Gamma_s$. Therefore, once we are able to resolve the sign ambiguity in $\Delta\Gamma_s$ by some other means, the $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 \phi$ decays are useful to measure $2\beta_s$ and 2γ without any ambiguity.

In order to resolve the sign ambiguity in $\Delta\Gamma_s$, and to reduce the discrete ambiguity in the measurement of $2\beta_s$ and γ , it is necessary to turn to Dalitz-plot analyses of three-body decays. We begin with $B_s^0(\bar{B}_s^0) \rightarrow D_s^\pm K^\mp \pi^0$. We find that it is possible to obtain $2\beta_s + \gamma$ with a twofold ambiguity, and to remove the sign ambiguity in $\Delta\Gamma_s$ (for this, it is not necessary to determine ϕ_s). The most promising method involves the decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$, in which all issues can be resolved. We find that $2\beta_s$ can be obtained without any ambiguity, and at the same we can remove the sign ambiguity in $\Delta\Gamma_s$. In addition, γ can be determined up to a twofold ambiguity.

Finally, all such decays allow the extraction of $\Delta\Gamma_s$ directly from a fit to the time-dependent untagged differential decay rate distribution. Given the measurements of ΔM_s , the semileptonic asymmetry a_{sl}^s , and $\Delta\Gamma_s$, the CP phase ϕ_s and $|\Gamma_{12}^s|$ can be obtained. In the case of three-body decays the coefficients of $\sinh[\Delta\Gamma_s t/2]$ and $\cosh[\Delta\Gamma_s t/2]$ can be found, either fully or partially, from a fit to the time-dependent tagged differential decay rate distribution. (Of the several three-body decays that we discuss, the decays $B_s^0(\bar{B}_s^0) \rightarrow D_{CP}^0 K \bar{K}$ are the most promising, since in such decays these coefficients can be fully reconstructed from this fit.) Therefore, in three-body decays the only unknown in the untagged rate distribution is $\Delta\Gamma_s$. This makes the fit considerably simpler.

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